

Dr. Kamlesh Kumar
Dept. of mathematics

①

date
12/06/24

M.Sc. Sem II, Paper-IX

Topology

Connectedness - Let X be a topological space. Then X is said to be dis-connected if there exist two (Non-empty, disjoint open sets) G and H of X such that $X = G \cup H$.

Now, $X = G \cup H$ and G and H are disjoint
 $\Rightarrow G \cap H = \emptyset$

So, $G = H^c$ and $H = G^c$

If G and H are open, they are closed also. Thus, a topological space X is said to be connected if X is not dis-connected.

Theorem:- A subset Y of a topological space X is disconnected iff Y is contained in the union of two open subsets G, H of X whose intersections with Y are non-empty and disjoint.

Proof:- we know
 $Y \cap (G \cup H) = (Y \cap G) \cup (Y \cap H)$

$\therefore Y \subseteq G \cup H \Leftrightarrow Y = (Y \cap G) \cup (Y \cap H)$ — ①

let Y be disconnected.

$\Rightarrow Y$ is union of two non-empty disjoint open sets, sets which are open for induced topology.

since, An open set of the induced topology is of the form $Y \cap G$ where G is open in X .

$\Rightarrow \exists$ two open subsets G, H of X such that $Y = (Y \cap G) \cup (Y \cap H)$ — (2)

such that $Y \cap G$ and $Y \cap H$ are non-empty and disjoint.

Eqn. (2) $\Rightarrow Y \subseteq G \cup H$, proved

to prove Y is disconnected

from (1)

$$Y = (Y \cap G) \cup (Y \cap H) \text{ — (3)}$$

\Rightarrow (3) is a disconnection of Y

$\Rightarrow Y$ is disconnected.

* A topology on a set X is a collection \mathcal{J} of subsets of X having the following properties:

(i) \emptyset and X are in \mathcal{J} .

(ii) The union of the elements of any subcollection of \mathcal{J} is in \mathcal{J} .

(iii) The intersection of the elements of any finite subcollection of \mathcal{J} is in \mathcal{J} .

A set X for which a topology \mathcal{J} has been specified is called a topological space.

If X is a topological space with topology \mathcal{J} , we say that a subset U of X is an open set of X . If U belongs to the collection \mathcal{J} .

If X is any set, the collection of all subsets of X is a topology on X , it is called the discrete topology.